GEOMATICS ENGINEERING DEPARTMENT

SECOND YEAR GEOMATICS

GEODESY 2 (GED209)

LECTURE NO: 10

ESTABLISHING OF BEST FITTING ELLIPSOID

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Overview Of Previous Lecture

- What is Height?
- What is A Height System?
- Vertical Datum
- Common Height Systems
- Height Determination Using Gravity Observations
- International Height Reference System (IHRS)
- Online Resources

Overview Of Today's Lecture

- Definition of Geodetic Datum
- What is meant by "Best Fitting"?
- Best Fitting Datum and how to achieve it in practice
- Notes on Establishment of Best Fitting Datum
- Significance of Accurate Geodetic Datum
- Summary

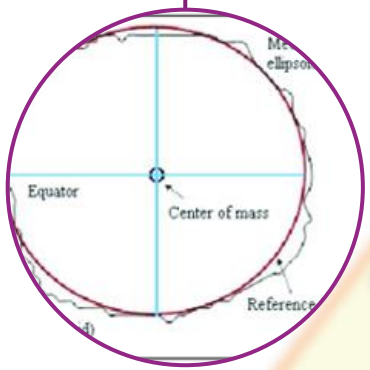
Expected Learning Outcomes

- Learn about geodetic datum
- Identify different types of datums.
- Gain knowledge about the main steps of establishing a local best fitting datum.
- Learn about some major applications of geodetic datums.

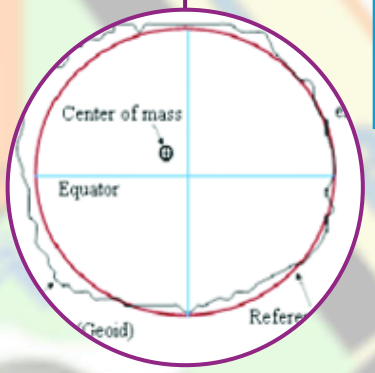
DATUM

Definition of Geodetic Datum

Datum



Geocentric



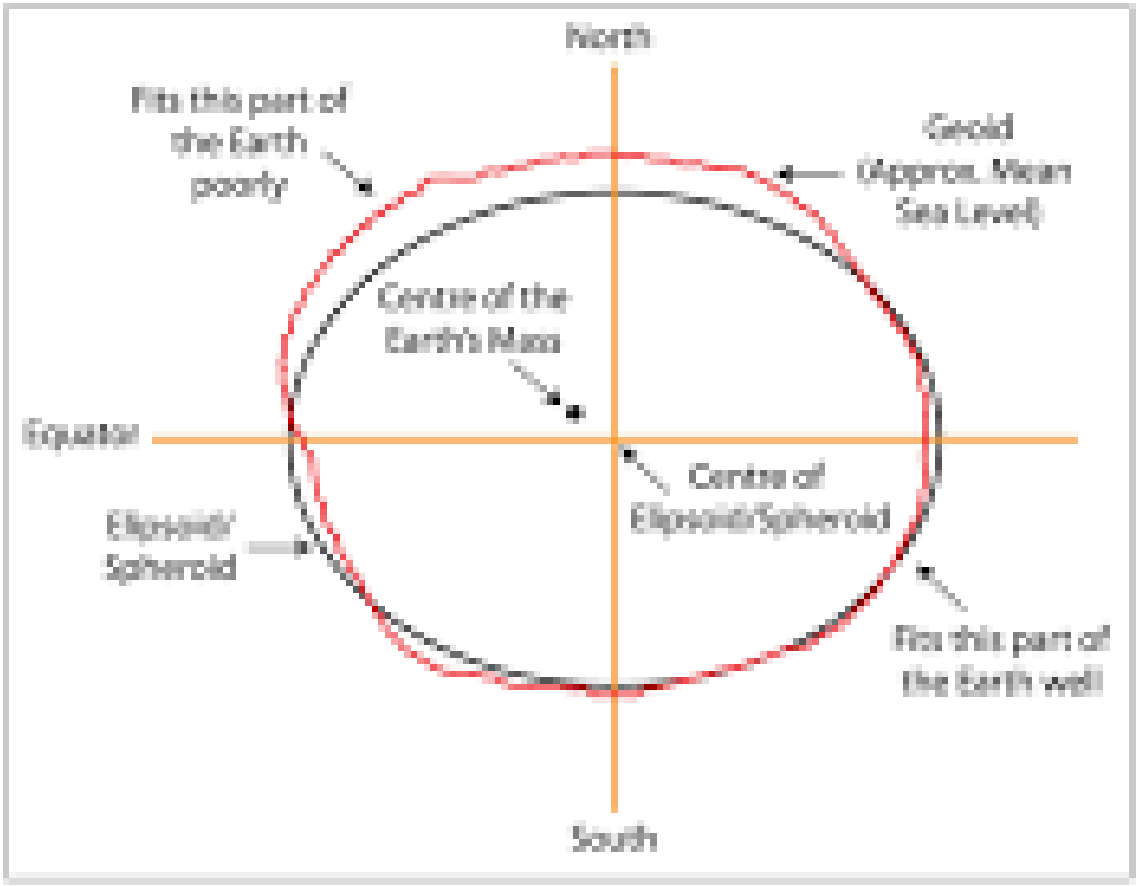
Topocentric

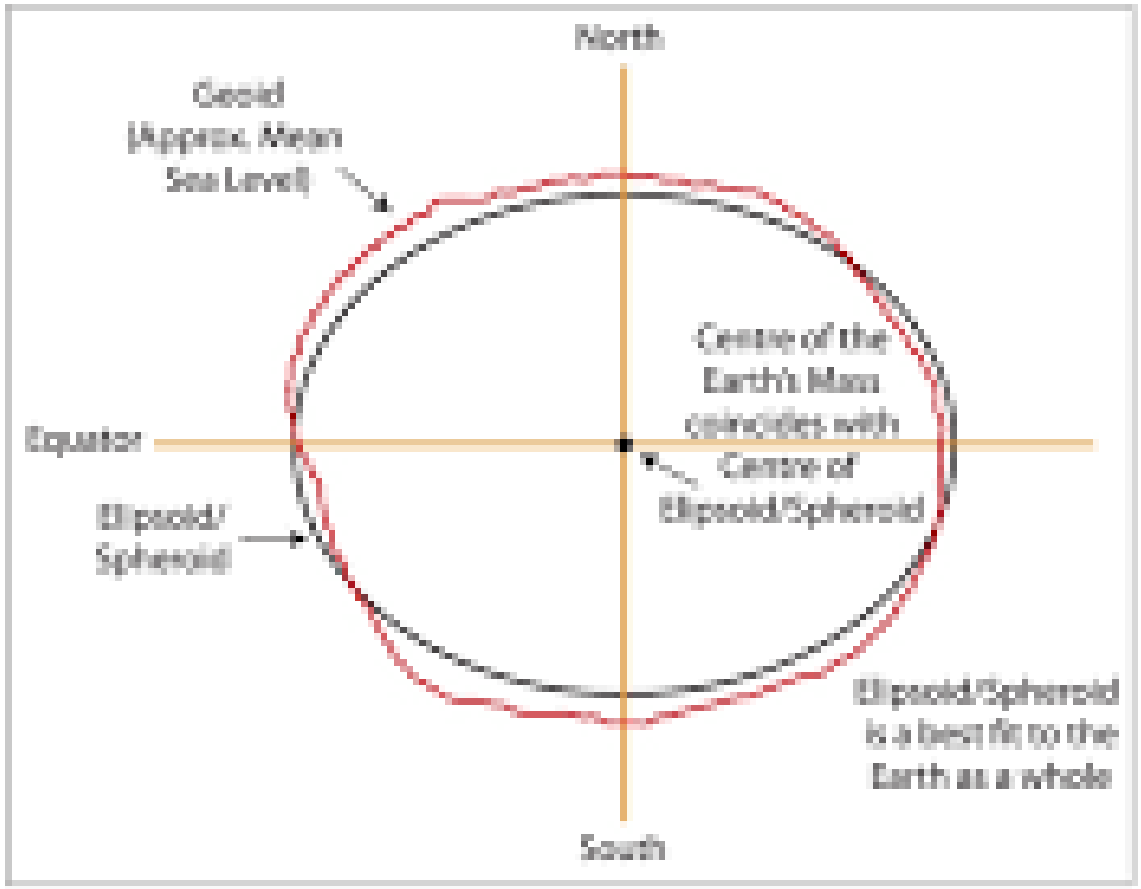
approximates the geoid as much as possible in the particular region of interest.

In this lecture, horizontal datum will be discussed in the context of best-fitting.

Definition of Geodetic Datum

- Learn

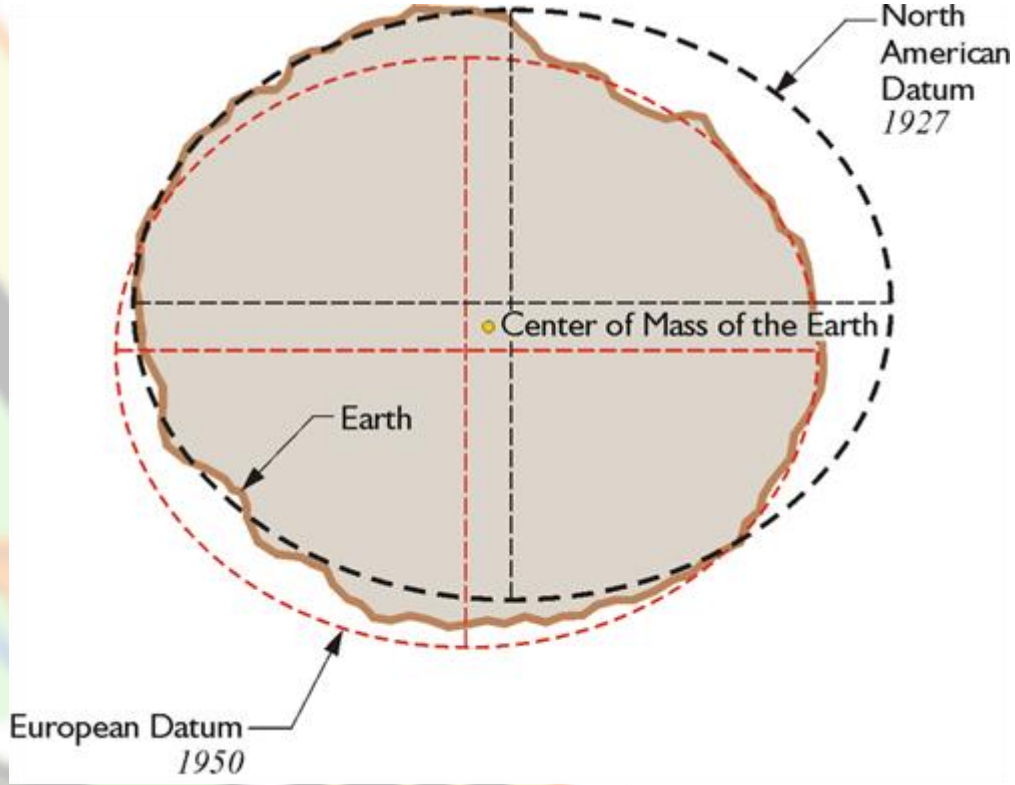




What is Meant by "Best-Fit"?

Best Fitting Geodetic Datum

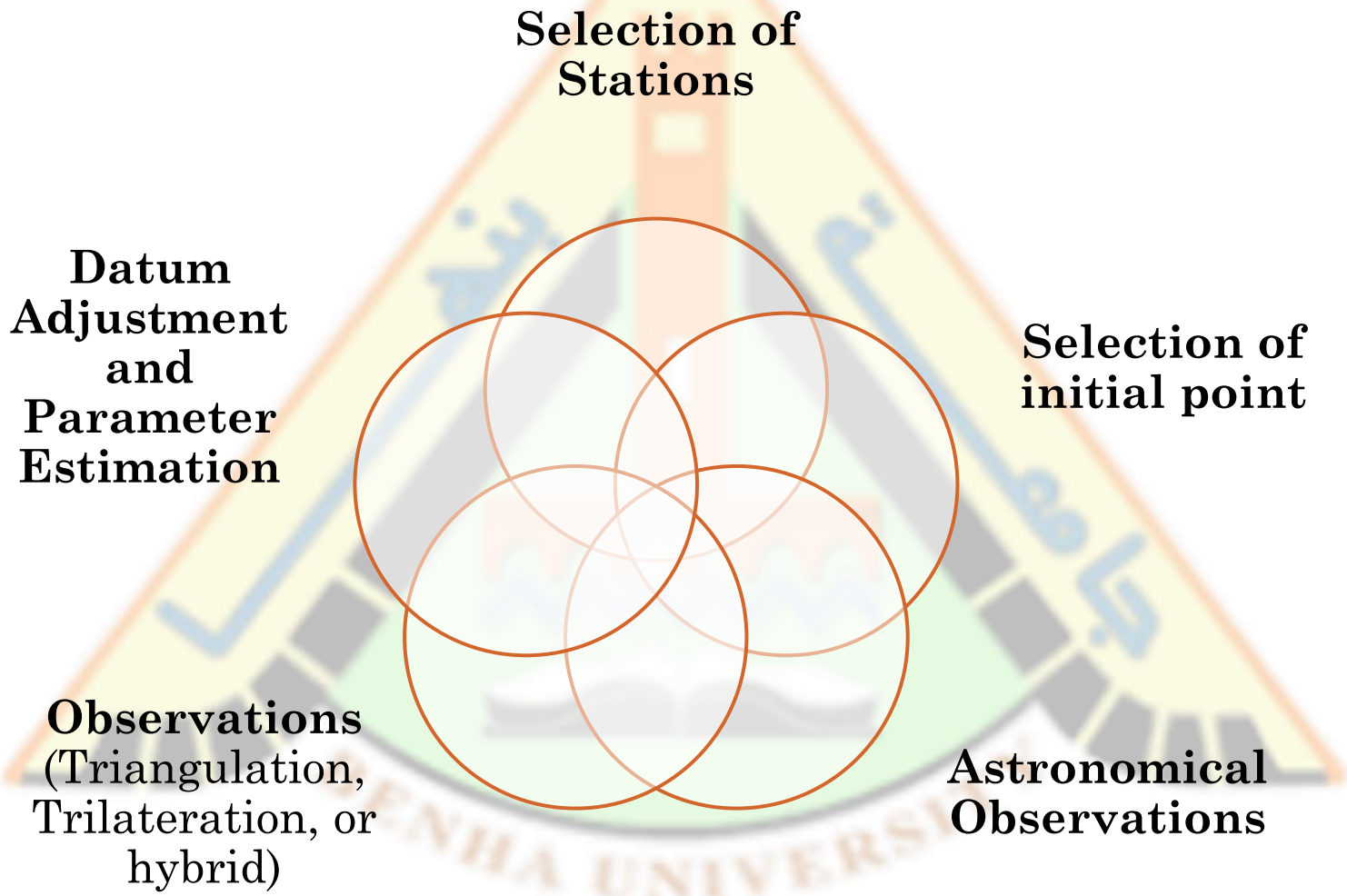
- The main objective of using a regional datum is to get minimum deviations between the geoid and reference ellipsoid over the area in question.
- When we achieve such an objective, we say that we have obtained a "best fitting ellipsoid" or a "best fitting datum", e.g., for our country.
- An ellipsoid that fits the geoid very well in a certain country does not necessarily fit in other country.



Best Fitting Datum and How To Achieve It In Practice

Astro-Geodetic Datum

Best Fitting Datum – Flowchart



Best Fitting Datum

- **Assumptions**

The problem of determining the datum positional parameters at the initial point is solved temporarily by assuming the ellipsoid and geoid to be tangent at the initial point as a preliminary orientation and use the astronomic observations to fix the other parameters at the initial point.

The Monument at the Initial Point of NAD27

Best Fitting Datum – Summarized Steps

1) Select the position of the datum initial point "i" (starting point of the network) to be in the *geometrical center* of the region of interest, and having a rigid terrain surrounded by areas of modest variations in gravity.

2) Select a reference ellipsoid, among the large list of ellipsoids used in practice, and specify the values of two parameters defining its size and shape (*e.g.* a and f).

3) Perform the preliminary orientation of the selected ellipsoid at the datum initial point, by setting : $\xi_i = \eta_i = N = 0$, and use the astronomic measurements to determine the geodetic coordinates of the initial point, as well as the geodetic azimuth of one initial line, i.e. $= \phi_i = \Phi_i$, $\lambda_i = \Lambda_i$ and $\alpha_{ij} = A_{ij}$.

BEST FITTING DATUM – SUMMARIZED STEPS

4) Form the observation equations for directions ,azimuths and distances for the network (taking the appropriate weights of observations into account), and perform a least-squares rigorous adjustment ending-up with the adjusted values of the network coordinates φ and λ.

5) Measure the astronomic latitude Φ_k and astronomic longitude Λ_k at all points "k" of the network, i.e. $k = 1, 2, \ldots, n$ where n is the number of points in the network. Then, using these measurements and their corresponding geodetic coordinates, compute the astro-geodetic geoid (i.e., the deflection components ζ, η, N).

6) Select one of the conditions of minimizing the deviations between the reference ellipsoid and the geoid as follows: -

Best Fitting Datum – Summarized Steps

$$\sum_{k=1}^{n} \acute{N}_k^{+} = 0; \quad (1)$$

$$\sum_{k=1}^{n} \acute{N}_k^{+2} = minimum; \quad (2)$$

$$\sum_{k=1}^{n} \theta_k^2 = minimum; \quad (3)$$

$$\sum_{k=1}^{n} (\xi_k^2 + \eta_k^2) = minimum; \quad (4)$$

BEST FITTING DATUM – SUMMARIZED STEPS

7) Denote the observed (i.e. computed) astro-geodetic deflection components, obtained from geodetic and astronomic coordinates, by just ζ_k and η_k, we can write the following expressions:

$$\bar{\zeta_k} = \zeta_k + d\zeta_k \quad (5)$$

$$\bar{\eta_k} = \eta_k + d\eta_k \quad (6)$$

where ξ_k and $d\xi_k$ are the changes required to be applied to the observed deflections (ξ_k, η_k) to make the sum of their squares a minimum and provide a best fitting ellipsoid. These changes, e.g. $d\xi_k$, $d\eta_k$ can be expressed as a function whose main arguments are the required changes in the ellipsoid size and shape parameters (a, f) and the independent three positional parameters (ξ_i, η_i, N_i^*) which were incorrectly specified to be zeros at the datum initial point. Such a function can be simply expressed as follows:

$$d\xi_k = F1(d\xi_i, d\eta_i, dN_i^*, da, df) \quad (7)$$

and:

$$d\eta_k = F1(d\xi_i, d\eta_i, dN_i^*, da, df) \quad (8)$$

BEST FITTING DATUM – SUMMARIZED STEPS

8. We can write these two equations for $d\xi_k$ and $d\ \eta_k$ For all stations having observed astro geodetic deflections "k", $k = 1, 2 \ldots\ldots n$, , in matrix notation as:

$$V = AX + L\ , \quad \text{(9)}$$

where

- V: is the vector of deflection components after minimization.
- L: is the vector of astro geodetic deflection components before minimization (i.e. computed from astro-observations.
- X :is a vector of five unknown components which are the two corrections to the chosen ellipsoid (da, df) and three corrections to the assumed deflection components and geoid undulation at the datum initial point ($d\xi_i$, $d\eta_i$, dN).
- A :is known as the coefficient matrix of the unknown parameters X.

Best Fitting Datum – Summarized Steps

9. Apply the parametric least-squares estimation procedure on Equation 9, which can be considered as an observation equation. The least-squares condition in this case will be:

$$V^T PV = minimum, \quad (10)$$

where P is the weight matrix of the observed astro-geodetic deflection components.

Equation 10 satisfies Equation 4 which is the condition for getting a best-fitting ellipsoid.

BEST FITTING DATUM – SUMMARIZED STEPS

10. Substituting from Equation 9 into Equation 10 and perform the minimization process, we finally end-up with the following solution vector X of the required corrections to the five previously stated unknown parameters, which is :

$$X = (ATPA)^{-1}(ATPL) \quad \text{(11)}$$

Add the components of X-vector, as obtained from the last equation, to the assumed approximate values of the five parameters, and get the new best fitting values for (ϕ_i, λ_i, N_i) of the initial point, as well as a, f.

Significance of Accurate Geodetic Datum

Significance of Accurate Geodetic Datum

- Foundation for Precise Positioning (Surveying and Mapping, Navigation, and Remote sensing)
- Geodynamics and Plate Tectonics
- Sea Level Monitoring
- Climate Change Research
- Facilitating Infrastructure Development
- International Collaboration and Data Sharing
- Economic Benefits (Efficient Land Management, Resource Exploration, and Improved Infrastructure Investment)

End Of Presentation

Thank You For Attention!